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# Raychaudhuri's Equation and the Present Universe in Brans–Dicke Theory

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Raychaudhuri's equation for the Brans–Dicke theory is used to relate $\dot{G}/G$ to the deceleration and the density parameters.

From Raychaudhuri's (1979) equation for the Brans–Dicke (1961) theory one can write, for the present universe, assumed pressureless, without acceleration, rotation, shear, or vorticity, that

$$\theta_{,0}+\frac{\theta^2}{3}+4\pi G(\rho+4\rho_\lambda)=0 \tag{1}$$

where

$$\theta=3H \qquad (H \text{ is Hubble's parameter}) \tag{2}$$

$$\rho_\lambda=\frac{2w+3}{32\pi G}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{3}$$

where $w$ is the coupling constant and $\phi$ is the scalar field, given by

$$\phi=aG^{-1} \qquad (a=\text{const}) \tag{4}$$

At least approximately, we can consider that, for the present universe, the deceleration parameter $q$ is constant. For such a case, it can be shown

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that (Berman, 1983)

$$H=\frac{1}{(1+q)}\,t^{-1} \tag{5}$$

Then, we can write for equation (1),

$$3H^2\left(-q+\frac{\Omega}{2}\right)=\frac{2w+3}{2}\left(\frac{\dot{G}}{G}\right)^2 \tag{6}$$

where

$$\Omega=\frac{\rho}{\rho_{\text{crit}}} \tag{7}$$

$$\rho_{\text{crit}}=\frac{3H^2}{8\pi G} \tag{8}$$

We can also assume on experimental grounds that

$$\frac{2w+3}{2}\approx w \tag{9}$$

Then we find

$$\frac{\dot{G}}{G}=\sigma H \tag{10}$$

where

$$\sigma=\pm\left[\frac{3}{w}\left(-q+\frac{\Omega}{2}\right)\right]^{1/2} \tag{11}$$

Will (1987) has reviewed the experimentally measured values for $\sigma$. The results are inconclusive. We see from (11) that $\sigma=0$ only if

$$q=\frac{\Omega}{2} \tag{12}$$

The result (11) is the real essence of what Raychaudhuri's equation for the Brans–Dicke theory can tell us about the present universe. The result (12) is the corresponding message that we get from Raychaudhuri's equation for general relativity theory.

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